Free spin quantum computation with semiconductor nanostructures

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Taking the excess electron spin in a unit cell of semiconductor multiple quantum-dot structure as a qubit, we can implement scalable quantum computation without resorting to spin-spin interactions. The technique of single electron tunnelings and the structure of quantum-dot cellular automata (QCA) are used to create a charge entangled state of two electrons which is then converted into spin entanglement states by using single spin rotations. Deterministic two-qubit quantum gates can also be manipulated using only single spin rotations with help of QCA. A single-short read-out of spin states can be realized by coupling the unit cell to a quantum point contact.

The idea of using electron spins in semiconductor quantum dots as qubits[1] has received tremendous attention in the implementation of scalable quantum computation. Recent experiments showing unusually long spin decoherence time in semiconductors[2, 3] provide a strong support for pursuing this idea. Up to date, several quantum computation schemes based on electron spins have be proposed with tunable Heisenberg type spin-spin interactions in semiconductor nanostrcutures[4, 5, 6, 7]. However, achieving a tunable spin-spin interaction with sufficient strengthes (comparing to the Coulomb interaction) is technically difficult. An interaction free mechanism for logical operations on electron spins is therefore more desirable. A few years ago, Knill, Laflamme and Milburn show in an influential paper [8] that quantum computation can be implemented with photons using only linear optics operators and single-photon detectors with feedback. The situation is quite different for free fermions according to the no-go theorem[9, 10]. Very recently, Beenakker et al. [11] show that for free flying fermions, one is able to construct a CNOT (controlled NOT) gate using only spin beam splitters and single spin rotations if charge detectors are added.

In this letter, we shall propose an implementation of scalable spin quantum computation without resorting to spin-spin interactions. We use external electrodes to control single electron tunnelings that create naturally a charge entangled state of two electrons with the help of a multiple-quantum-dot structure, the quantum-dot cellular automata (QCA). The charge entangled state is then converted into a spin entangled state of the electrons using only single spin rotations (we call it as charge-to-spin convention of electron entanglement states). Spin-spin interactions are not required in this implementation and deterministic two-qubit controlled gates can be easily manipulated. Thus, a free spin quantum computation is feasible with semiconductor nanostructurs.
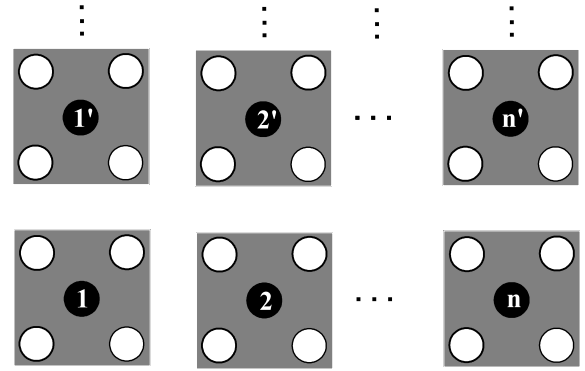
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FIG. 1: Schematic of a scalable free spin quantum computer based on semiconductor nanostructures. Each shading square box (contains five quantum dots) is taken as a unit cell.

The architecture of our scalable quantum computer is based on the semiconductor multiple quantum dot structures schematically shown in Fig. 1. The basic devices (the shading boxes in Fig. 1) are considered as unit cells. Each cell contains a qubit quantum dot (the central black dot) surrounding with four ancilla dots (the empty dots). The detailed structure of the cell i is given by Fig. 2a. The lines between quantum dots in a cell indicate the possibility of interdot tunneling. The energy barriers between the neighboring cells must be high enough to ensure that tunnelings of electrons between different cells are forbidden. We also assume that each cell is charged with only one excess conductor electron and each dot in the cell is considered as a site. The on-site charge energy E_0 of the excess electron in the qubit dot is low enough comparing with the on-site energy E_a in the ancilla dots ($\varepsilon = E_a - E_0 > 0$) such that the excess electron will sit initially in the qubit dot due to the Coulomb blockade effect (see Fig. 2b). Furthermore, the four ancilla dots within the cell are coupled to the qubit dot through bias electrodes such that by tuning on the bias voltage V_i^{LR} or V_i^{TB} (the anti-bias voltage is given by $-V_i^{LR}$ or $-V_i^{TB}$), the excess electron will tunnel coherently into the right or

the bottom (the top or the left for an anti-bias voltage) two ancilla dots with equal possibility if the ancilla dots are fabricated identically (also see Fig. 2c). This architecture can be achieved by current/further development of nano-technology.
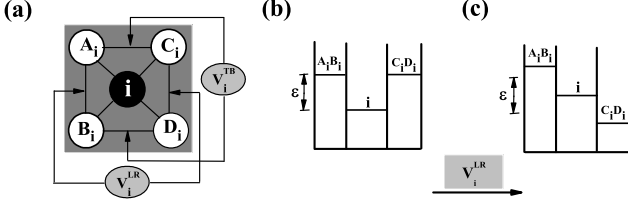

FIG. 2: (a) The detailed structure of the unit cell i. The bias electrode voltages V_i^{LR} and V_i^{TB} acting on the cell control the electron tunnelings among dots within the cell; (b) and (c) show the on-site energy of the electron in different quantum dots in the cell without and with the bias voltage V_i^{LR} respectively.

Based on such an architecture of the basic quantum devices, the four empty dots between two qubit dots of the neighboring cells (given by the dotted square box, e.g. C_i-D_i-B_j-A_j in Fig. 3) form a usual structure of QCA [12]. QCA has been used to simulate classical digital algorithms [13]. A semiconductor realization of such a structure has also been developed[14]. Quantum mechanically, when a QCA is charged with two electrons, these two electrons will occupy coherently two diagonal sites (two charge polarizations) as a result of Coulomb repulsion[15]. The corresponding two-electron charge state can be in an arbitrary superposition state of the two charge polarizations ($P = \pm 1$) which creates indeed an entangled charge state of the two electrons. In additional, an external bias polarization E_{bias} is coupled to each QCA for adjusting the splitting of two charge polarizations[15], as shown in Fig. 3.
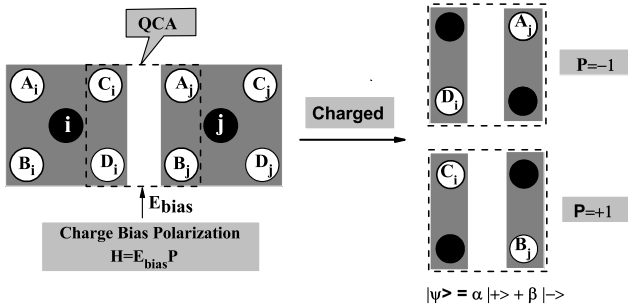

FIG. 3: Quantum mechanically, the four quantum dots (the dotted square boxes) between two qubit dots of neighboring cells form a coherent QCA.

We shall use this quantum mechanical QCA structure to manipulate electron spin entangled states and two-qubit control gates through single electron tunnelings and single spin rotations only. To be explicit, we define quantum states of the excess electron in each cell as a

direct product of electron spin and charge states $|S_i\rangle|e_i\rangle$ ($i = 1, 2...$), where the electron sits initially in the qubit dot. The spin states of the excess electron $|S_i\rangle$ are chosen as the qubit states in Pauli basis $|\uparrow\rangle = |0\rangle$ and $|\downarrow\rangle = |1\rangle$, the electron charge state $|e_i\rangle$ is considered as an ancilla state. A static uniform magnetic field is applied to split the qubit states $|0\rangle$ and $|1\rangle$ by Zeeman energy. The excess electron in each cell can be driven away from its initial site (the qubit dot) in ancilla dots only when a two-qubit controlled operation is performed, and will be pushed back to the initial site as soon as the two-qubit operation is completed, this is controlled by the bias voltage pulses V_i^{LR}, V_i^{TB}. To distinguish the different sites of the electron in the cell, we denote the charge state of the electron siting in the ancilla dots A_i, B_i, C_i, and D_i by $|e_i^X\rangle$ with $X = A, B, C$ and D, respectively. The site dependence of electron spin state is ignorable.

Now, two-qubit controlled operations can be implemented as follows: consider a pair of neighboring unit cells, e.g. the i-th and the j-th cells in Fig. 3. The initial state of the two excess electrons is given by

$$|\Psi_0\rangle = |S_i S_j\rangle|e_i e_j\rangle, \qquad (1)$$

By tuning on the bias voltage V_i^{LR} and the anti-bias voltage $-V_j^{LR}$ to lower the electron potential energy of the dots C_i, D_i and A_j, B_j, the excess electron in each cell is tunneled with definite probabilities into the quantum dots C_i, D_i and A_j, B_j, respectively. If we assume that the ancilla dots in each cell are identical, the tunneling rates of the excess electron into the dots C_i and D_i (A_j and B_j) in the cell i (j) are equal. As mentioned before, the four quantum dots (C_i, D_i, A_j, B_j) form quantum mechanically a coherent QCA. When the QCA is charged with two excess electrons, these two electrons will occupy coherently two diagonal sites as a result of Coulomb repulsion. Since the electron tunnelings between different cells are forbidden, the two electron charge state becomes a superposition state of the two charge polarization states,

$$|\Psi_0\rangle \overset{(V_i^{LR}, -V_j^{LR})_{\text{on}}}{\longrightarrow} |\Psi_1\rangle = |S_i S_j\rangle \frac{1}{\sqrt{2}} \left(|e_i^C e_j^B\rangle + |e_i^D e_j^A\rangle \right). \qquad (2)$$

It shows that with the help of QCA, the two electron charge state becomes a maximally entangled state after switching on the bias voltages V_i^{LR} and $-V_j^{LR}$.

Now, we can convert the charge entangled state into a spin entangled state using only single spin rotations. Explicitly, consider the initial spin state of the two electrons: $|S_i S_j\rangle = |\uparrow\downarrow\rangle = |01\rangle$. We take a spin rotation $R_x(\pi)$ on each electron sited at the dots D_i and A_j respectively after Eq. (2), where $R_x(\theta) \equiv \exp(-i\theta\sigma_x/2)$. The corresponding spin state of the two electrons becomes $R_x^D(\pi)R_x^A(\pi)|01\rangle = -|10\rangle$. Following the rotation operations, we push the two electrons back into the qubit dots i and j by switching off the bias voltages V_i^{LR} and $-V_j^{LR}$. Thus, the charge states return back to the initial states: $|e_i^C e_j^B\rangle \rightarrow |e_i e_j\rangle$ and $|e_i^D e_j^A\rangle \rightarrow |e_i e_j\rangle$. Conse-

quently, we have

$$|\Psi_1\rangle \xrightarrow{R_x^D(\pi)R_x^A(\pi)} \frac{1}{\sqrt{2}}\big(|01\rangle|e_i^C e_j^B\rangle - |10\rangle|e_i^D e_j^A\rangle\big)$$

$$\xrightarrow{(V_i^{LR},-V_j^{LR})_{\text{off}}} \frac{1}{\sqrt{2}}\big(|01\rangle - |10\rangle\big)|e_i e_j\rangle, \qquad (3)$$

namely, the electron charge entangled state has been converted into a spin Bell state $|\psi^-\rangle$. Here, tuning on and tuning off the bias voltages V_i^{LR} and $-V_j^{LR}$ can be achieved by bias electrode pulses[17], the single spin rotations can be implemented by either local magnetic fields or ultrafast optical pulses as we shall discuss later. The pulse sequence for generating the above spin Bell state through QCA is illustrated in Fig. 4a. Repeating the process of Eqs. (1-3) with different initial spin states and single spin rotations, we can generate other three spin Bell states ($|\psi^+\rangle, |\phi^-\rangle, |\phi^+\rangle$):

$$|01\rangle|e_i e_j\rangle \xrightarrow{R_x^D(\pi)R_x^A(3\pi)} \frac{1}{\sqrt{2}}\big(|01\rangle + |10\rangle\big)|e_i e_j\rangle,$$

$$|00\rangle|e_i e_j\rangle \xrightarrow{R_x^D(\pi)R_x^A(\pi)} \frac{1}{\sqrt{2}}\big(|00\rangle - |11\rangle\big)|e_i e_j\rangle, \quad (4)$$

$$|00\rangle|e_i e_j\rangle \underbrace{\xrightarrow{R_x^D(\pi)R_x^A(3\pi)}}_{V_i^{LR},-V_j^{LR}} \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big)|e_i e_j\rangle.$$
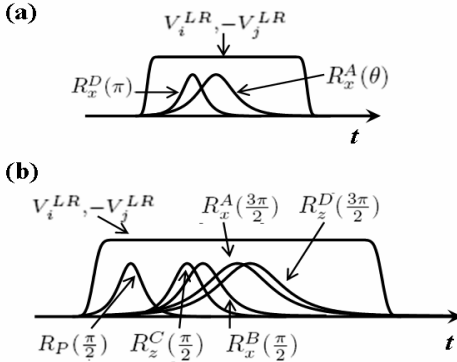

FIG. 4: (a) An illustration of the pulse process for creating spin Bell states through QCA, where $\theta = \pi$ or 3π; (b) The pulse process for CNOT gate.

To construct a two-qubit controlled gate, we shall modify first the charge entangled state in Eq. (2) with the help of the bias polarization E_{bias}. The charge polarizations of two-electron states in QCA are defined as $|e_i^C e_j^B\rangle = |-\rangle$ and $|e_i^D e_j^A\rangle = |+\rangle$ with polarizations $P = \mp 1$. The bias polarization E_{bias} coupling to QCA results in an effective Hamiltonian $H_P = E_{\text{bias}}\hat{P}$ [15], where $\hat{P}|+\rangle = |+\rangle$ and $\hat{P}|-\rangle = -|-\rangle$. Thus, applying a bias polarization $\pi/2$-pulse to Eq. (2), we obtain

$$|\Psi_1\rangle \xrightarrow{R_P(\frac{\pi}{2})} e^{i\pi/4}|S_i S_j\rangle\frac{1}{\sqrt{2}}\big(|e_i^C e_j^B\rangle - i|e_i^D e_j^A\rangle\big). \quad (5)$$

Then, we apply further a single spin rotation on each dot C_i, D_i, A_j, B_j in the QCA with the operators $R_z^C(\frac{\pi}{2})$, $R_z^D(\frac{3\pi}{2})$, $R_x^A(\frac{3\pi}{2})$, $R_x^B(\frac{\pi}{2})$ respectively to rotate the corresponding electron spin states. Finally, by switching off the bias voltages V_i^{LR} and $-V_j^{LR}$, the two excess electrons return back to the qubit dots in the cells i and j. With these operations, the initial state Eq. (1) becomes

$$\begin{array}{ccc} |00\rangle|e_i e_j\rangle & & |00\rangle|e_i e_j\rangle \\[4pt] |01\rangle|e_i e_j\rangle & \xrightarrow{U_P U_S(ABCD)} & |01\rangle|e_i e_j\rangle \\[4pt] |10\rangle|e_i e_j\rangle & \underbrace{\phantom{\xrightarrow{U_P U_S(ABCD)}}}_{V_i^{LR},-V_j^{LR}} & |11\rangle|e_i e_j\rangle \\[4pt] |11\rangle|e_i e_j\rangle & & |10\rangle|e_i e_j\rangle \end{array} \qquad (6)$$

where, $U_P \equiv R_P(\frac{\pi}{2}), U_S(ABCD) \equiv R_x^A(\frac{3\pi}{2}) \otimes R_x^B(\frac{\pi}{2}) \otimes R_z^C(\frac{\pi}{2}) \otimes R_z^D(\frac{3\pi}{2})$. Thus, a two-spin qubit CNOT gate is manipulated using only single spin rotations through QCA for single electron tunnelings. The pulse sequence for such an manipulation is given in Fig. 4b.

The above discussion shows that the spin two-qubit controlled operations can be achieved using only single spin rotations and single electron tunnelings through QCA. The method of using bias electrodes to control single electron tunnelings has been investigated in recent years for single electron transistor (SET) devices [16] and also for the manipulation of single electron charge qubit in double quantum dots[17] at a time scale of a few hundred picosecond. Using bias electrodes to control electron charge polarizations in QCA has also been proposed and discussed[15]. The single spin rotations in semiconductor quantum dots have been extensively explored[2]. The simplest manipulation of a single spin is to expose individual dots to a time-varying Zeeman coupling $(g\mu_B\mathbf{S} \cdot \mathbf{B})(t)$, which is controlled through the local magnetic field $\mathbf{B}$ or the local g-factor in semiconductor nanostructures. Localized magnetic fields can be generated with the magnetic tip of a scanning force microscope. The local g-factor can be modified by external bias voltage. An effective Zeeman field may also be realized by exchange spin coupling to ferromagnetic dots [1], but spin exchange couplings for single spin rotations are not our choice for implementing free spin quantum computation.

Considering the problem relating to decoherence in semiconductor nanostructures, we prefer to use another method for the manipulation of single electron spins, i.e., the fast controls of single spin coherence using ultrafast optical pulses. It has been experimentally demonstrated that optical tipping pulses with a frequency below the band gap of the semiconductor nanostructures can create an effective magnetic field on the order of 20 T via the optical Stark effect, which can induce substantial rotations of electron spins at femtosecond scales[18]. Meanwhile, spin-flip Raman transitions using the adiabatic process of two ultrafast laser pulses[19] can also fully control single spin rotations in semiconductor quantum dots at picosecond or femtosecond scales[7, 20]. These optical controls

of single spin rotations are technically attractive for practical manipulation. As it has been demonstrated experimentally that the typical decoherence time of a electron spin in semiconductor nanostructures is about 50 μs, the bias voltage pulse for single electron tunnelings has the time scale of a few 100 ps. Fig. 4 tells us that the single spin rotations must be completed much faster than the bias voltage pulses. Thus, the ultrafast optical pulses for manipulating single spin at picosecond to femtosecond scales are required to reduce decoherence effects.

At last, we shall also discuss the initialization and read-out of single electron spins in this scheme. A static uniform magnetic field can be applied to split the spin up state $|0\rangle$ and the spin down state $|1\rangle$ by the Zeeman energy for initialization. A single-shot read-out of the electron spin states in qubit dot can be realized by coupling the unit cell to a quantum point contact (QPC). Explicitly, one can tune a bias voltage pulse, e.g. V_i^{LR}, to lower the on-site energy of dots C_i, D_i such that the electron will remain in the qubit dot if it is in the state $|0\rangle$, otherwise it will tunnel to the dots C_i, D_i and then return back to the qubit dot after the pulse if it is in the state $|1\rangle$. By measuring the charge current through the QPC channel, I_{QPC}, one can detect changes in charge that result from the electron tunneling between the qubit dot and ancilla dots. In this way, we can measure the single electron spin states in qubit dots through the QPC as a charge detector. Such a measurement has been experimentally realized in quantum dots[21].

In summary, combining the spin two-qubit CNOT gate with single spin rotations, a universal quantum computation can be achieved without using spin-spin interactions. Implementing free fermion quantum computation is a very challenge subject in principle[9, 10]. Here we are able to achieve such an implementation relying basically on charge-to-spin conversion through QCA. QCA offers an intrinsic charge coupling of two electrons, which is more effective than the use of beam splitters plus charge detectors[11]. Since spin exchange interaction is much weaker than electron Coulomb interaction (by the order of 10^{-3}), such an implementation of free spin quantum computation has the advantage of being robust against the technical difficulties of generating strong spin-spin interactions. We hope that the realization of this scheme will bring a new challenge to semiconductor spintronics and the development of nanotechnology.

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